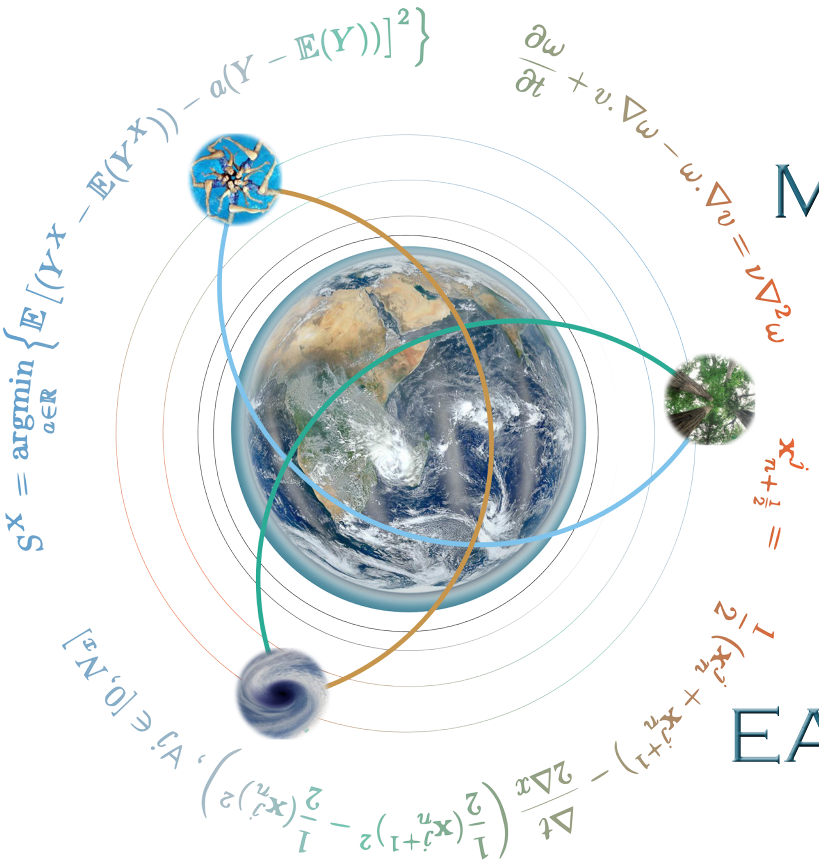
The Consortium MathsInTerre presents



MATHEMATICS & THE COMPLEXITY OF THE EARTH SYSTEM

The current state and future of French research into mathematics and the environment

A summary of the MathsInTerre 2013 Research Thinktank

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ALLenvi
French Alliance for Environmental Research

ALListene
Alliance for digital sciences and technologies

AMIES
Agency for Mathematics in Interaction with Enterprises and the Society

ANDRA
National radioactive waste management Agency

ANR
French National Research Agency

ARP
Research ThinkTank

CEA
French Alternative Energies and Atomic Energy Commission

CEMRACS
Centre d'Eté Mathématique de Recherche Avancée en Calcul Scientifique

CIRM
Centre International de Rencontres Mathématiques

CNRS
National Center for Scientific Research

CNU
National Council of Universities

EPA
Public establishment of an administrative nature

EPIC
Public establishment of an administrative nature

EPST
Public Scientific and Technical Research Establishment

ERC
European Research Council

FMSP
Foundation Sciences Mathématiques de Paris

GdR INSMI
Mathematical Sciences Research Groups of the CNRS

IFSTTAR
French Institute of Science and Technology for Transport, Development and Networks

IFREMER
French Research Institute for Exploitation of the Sea

IHÉS
Institut des hautes études scientifiques (Institute for advanced research in mathematics, theoretical physics)

IHP
Henri Poincaré Institute (International structure dedicated to mathematics and theoretical physics)

INED
French National Institute for Demographic Studies

INRA
French National Institute for Agricultural Research

INRIA
National Institute for Research in Computer Science and Control

INSERM
French National Institute of Health and Medical Research

INS2I
Institute for Information Sciences and Technologies, CNRS

INSHS
Institute for Humanities and Social Sciences, CNRS

INSMI
National Institute for Mathematical Sciences, CNRS

INSU
National Institute for Earth Sciences and Astronomy, CNRS

INEE
Institute of Ecology and Environment, CNRS

IPCC
Intergovernmental Panel on Climate Change

IRD
Institute for development research

IRSTEA
National Research Institute of Science and Technology for Environment and Agriculture

LEFE-MANU
French national program for mathematical and numerical methods for geophysical fluids, INSU-CNRS.

MaiMoSiNE
Maison de la Modélisation et de la Simulation, Nanosciences et Environnement

RNSC
French Network for Complex Systems

SFdS
French Society of Statistics's

SMAI
French Society for Applied and Industrial Mathematics

SMF
Mathematical Society of France

SHOM
Naval Hydrographic and Oceanographic Service

UFR
Training and research units, inside universities

« The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. We should be grateful for it and hope that it will remain valid in future research and that it will extend, for better or for worse, to our pleasure, even though perhaps also to our bafflement, to wide branches of learning. »

- Eugene Wigner, The Unreasonable Effectiveness of Mathematics in the Natural Sciences, Communications in Pure and Applied Mathematics, vol 13. No I, 1960.

Foreword

The 'MathsInTerre' Research Thinktank is a project financed by the French National Research Agency (ANR), hosted by the IHP and supported by the INSMI - CNRS, structures that realise the importance of reflecting on the subject of "Mathematics and the complexity of the Earth system". The project was launched in January 2013 for one year with the aim of making a contribution to a more systemic and integrated vision of the subject. Its ultimate goal is to stimulate discussion upstream of the definition of an action plan for the ANR. Beyond this, it is a question of providing input for analyses and debates in the scientific communities concerned by the issues associated with the rapprochement of mathematics and the Earth sciences.

The Thinktank's overriding mission was to construct a collective analysis from truly multidisciplinary interaction. Thus, to avoid the pitfall of adopting a viewpoint restricted to mathematics, we worked in collaboration with a wide consortium of researchers including mathematicians and scientists who were not from the mathematics community. We also benefited from collaboration with learned societies in mathematics, the INSMI-CNRS Research Groups, the AMIES and the RNSC.
Thanks to these numerous partnerships, the Thinktank was able to communicate and use the funding of the ANR to implement different formats for discussions and meetings. The variety of supports used enabled this extensive community to collectively reflect on the state of knowledge in the field and the collaborative actions to be developed as a priority.

The results of this reflection are now available in the form of a report. The present document is the written synthesis of this report, translated into english by Brian Keogh.

More information on the Thinktank is available on the site "mathsinterre.fr," managed by Stania Raitmayerova. Further information on this presentation can be found on the internet at http://bit.ly/1bUZWag.

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Acknowledgements

This document would not have been possible without the help and commitment of numerous researchers from a variety of disciplines that got involved in this project. We would like to thank them for all their work and their investment, particularly given the fact that there are many other demands on their time from an increasing number of administrative activities in the research field.

The Henri Poincaré Institute (IHP), created in 1928, is one of the oldest and most dynamic international structures dedicated to mathematics and theoretical physics.
The aim of the National Institute for Mathematical Sciences (INSMI), a branch of the CNRS, the French national scientific research centre, is to promote excellence in French mathematics by facilitating and coordinating a network of research units, both at the national and international levels.
It is important to stress the role of the IHP and INSMI in enabling us to carry out a collective project of the highest quality. In this respect, we would particularly like to thank the administrative personnel of the IHP (namely Marjorie Stievenart-Ammour, Brigitte Bony and Florence Lajoinie) and the DR 2 (Paris B) of the CNRS.

Didier Bresch (DR CNRS-INSMI),
Emilie Neveu (Post-doc IHP).

Introduction

Mathematics is a fundamental discipline that has always been at the heart of important issues relating to the complexity of the Earth and, more especially, the environment. Understanding research issues as well as problems of sustainable management requires the adaptation of mathematical techniques in interaction with other disciplines. However, seeking solutions to environmental problems can also give rise to the development of new mathematical theories. These essential exchanges between disciplines gradually lead to a better understanding of the complexity of the world we live in.

This complexity is reflected in the diversity of the topics studied: the applications to concrete problems, relating to the world that surrounds us, from genetic evolution to fluid turbulence. Emphasis is also put on mankind and his interaction with the ecosystem. We focused on the interactions between different processes and different scales by considering mathematics as a discipline that cuts across multiple fields of knowledge. Thus, the thinktank does not intend to be exhaustive but rather to demonstrate the diversity of mathematics' role by considering a number of carefully chosen examples.

Building bridges between mathematics and the other disciplines was one of the fundamental roles of the Thinktank

Throughout this synthesis, we make references to all the texts written by the different researchers. For those who wish to find out even more, the texts appear in their entirety in the report.

Within this wide-ranging field of interest, the MathsInTerre Thinktank organised different topics according to three themes: Human World, Living World, Fluid World (see insert below).

This synthesis, like the report, has been divided into three parts:

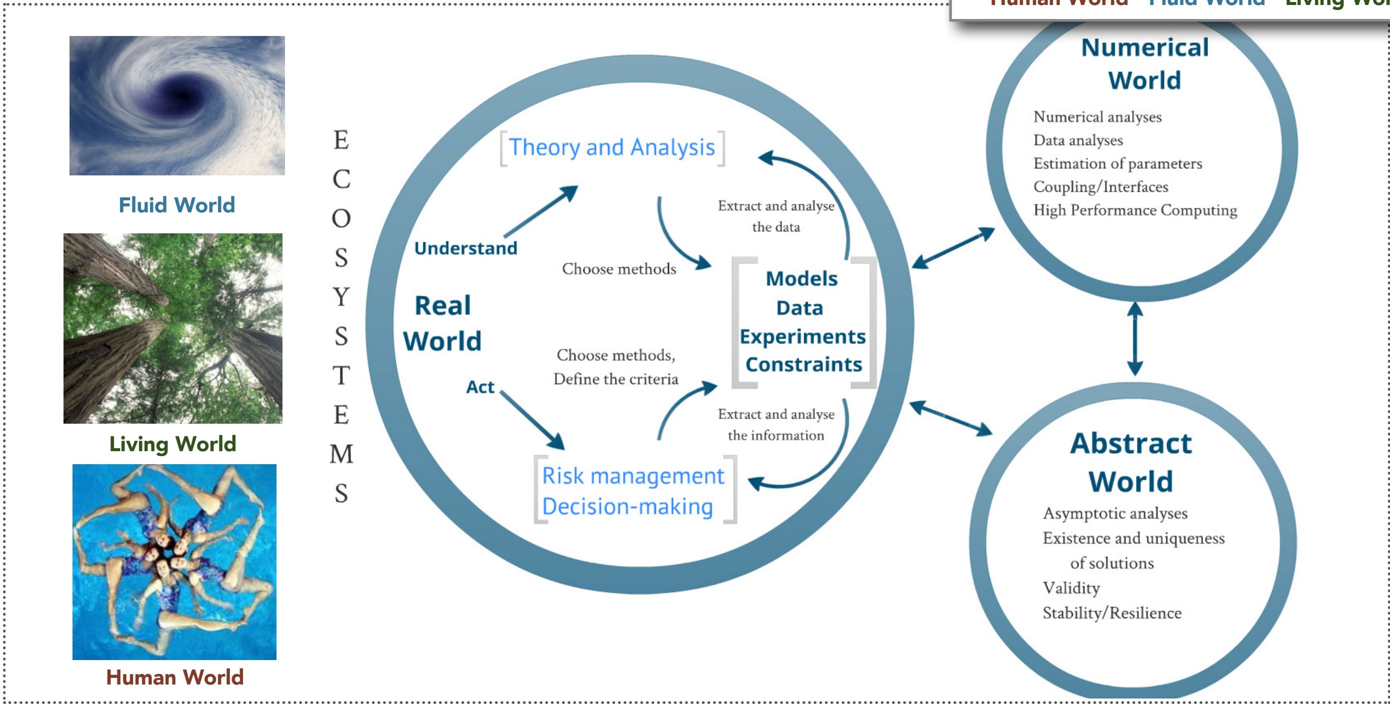
Mathematics in the real world focuses on the point of view of researchers, whether mathematicians or not, working on questions relating to the world we live in.

Emerging mathematics focuses on the point of view of researchers working on the development of new branches of mathematics on the basis of questions raised by the study of the planet Earth.

Numerical mathematics focuses on the point of view of researchers in numerical mathematics, who are often at the interface between theory and applications.

These three parts each present a different point of view, reflecting the diversity of mathematics. It goes without saying that the topics and themes are not independent of one another. This arbitrary division is a choice made for the sake of clarity but at the same time it also reflects the problems of relational boundaries between the different actors. This brings us to the final part, ***Proposals for structural action plans***, which presents a number of suggestions for concrete action that are likely to promote collaboration and the exchange of information between mathematics and the other disciplines. These proposals provided the basis for different discussions between concerned specialists and researchers. They were adopted unanimously by the consortium associated with the Thinktank.

DISCOVER THE REPORT BY FOLLOWING THE SIGNS:



Mathematics: A multi-present, cross-border discipline

During our discussions with non-mathematician communities, questions were often raised about mathematics that at first appeared naïve. Here, we would like to remind readers what mathematics is and what it does, basing our observations on the important work of learned societies and the initiative entitled "Mathematics of Planet Earth 2013".

"It's a discipline that feeds on its links with the other sciences, with society and the industrial world, but one that is also enriched itself in doing so (translation)."
- Mathématiques, l'explosion continue.

What does a mathematician do?

One of the mathematician's roles is to observe the world and try to create conceptual tools to better understand it. All the power of mathematics lies in this two-fold approach: sometimes it can be used to resolve concrete problems, sometimes it develops very abstract and general methods that are disconnected from the real world but that – sometimes – have concrete and unexpected consequences.
From the most abstract theories to the most concrete analytical methods, there is not a single mathematics but several! And they are all essential and intertwined.
We have heard a lot about the rigour of the discipline but few people realise that the mathematician is, at the same time, flexible and open-minded! Indeed, the mathematician extends and adapts each of his or her objects of study to other applications and benefits from the links woven between different researchers in mathematics and other disciplines (in France and internationally). These links enable the mathematician to solve new problems.

Inspired by the text in Le Cercle, Les Echos d'E. Ghys (UMPA ENS Lyon).

What can mathematics contribute to environmental issues?

Mathematics is essential in everyday life. Thus, environmental and social issues that require the analysis, optimisation and management of increasingly complex systems result in a growing need for mathematics. This important role is well understood by certain countries, such as China and the United States, a fact that has been reflected in increased financial support for the discipline in these areas.
Furthermore, the overall perception of the mathematician as being non-discriminatory and flexible enables him or her to play an important role in the dissemination of knowledge. "Mathematics … develops intuition, imagination, and a critical mind; it is also a universal language and a strong element of culture."

- Mathématiques, l'explosion continue.

Why work in a theoretical and abstract manner? It doesn't seem very useful!

It is this kind of typical remark that led to this page being written! It should be remembered that scientific progress in applied fields often comes about because of recent or past fundamental research (see *Mathematics and the Earth: a long story*, p.7).
It is important to keep in mind that "wishing to define activity or research in mathematics through its existing or potential applications would result in their disappearance. At the other end of the spectrum, favouring axiomatization, the study of structures and the internal dynamics of the discipline, as French mathematics did in the 1940-1970 period, albeit with considerable success, led to the development of so-called applied mathematics in France being held back *(translation)*" -Mathématiques, l'explosion continue.
Today many research mathematicians are involved in problems with concrete applications through exchanges and close collaboration with other disciplines. By way of example, mention may be made of initiatives such as the chair in Modélisation mathématique et biodiversité at the Polytechnique/MHN (Muséum national d'Histoire Naturelle) or the chair in Modélisation prospective at the CMA/ParisTech (Centre de Mathématiques Appliquées).
It is now important to focus on the Science-Earth link, treating it as a continuum, without any barriers between disciplines. It is questions arising from concrete issues that must determine the mathematics conducted in relation to the Earth, whether this is ***Mathematics in the real world***, ***Emerging mathematics***, or ***Numerical mathematics***. This question must become one of the essential criteria in determining the quality of applied mathematics.

further reading

UNESCO granted its patronage to a project entitled "***Mathematics of Planet Earth 2013***", which took on the mantle of an international year. Within this framework, a number of intelligent initiatives were taken with a view to making mathematics more accessible.

Mathématiques, l'explosion continue, is a brochure of some twenty texts presenting the contribution of mathematics in addressing questions of real concern in today's world. Distributed by the Foundation Sciences Mathématiques de Paris, the SMF, the SMAI, and the SFdS, with the help of Cap'Maths.

Un jour, une brève, mpt2013.fr, is an ambitious web site which, on the initiative of the SMAI, the SMF, the SFdS, the CNRS, the INRIA and Cap'Maths, published every day of the week in 2013 a new example of the application of mathematics in our society and for the environment. Something to make you appreciate the need for even the most theoretical research!

Images des mathématiques (images.math.cnrs.fr) and ***Interstices*** (interstices.info) did not wait for 2013 to inform as wide a public as possible about mathematics research.

And what's it like elsewhere?

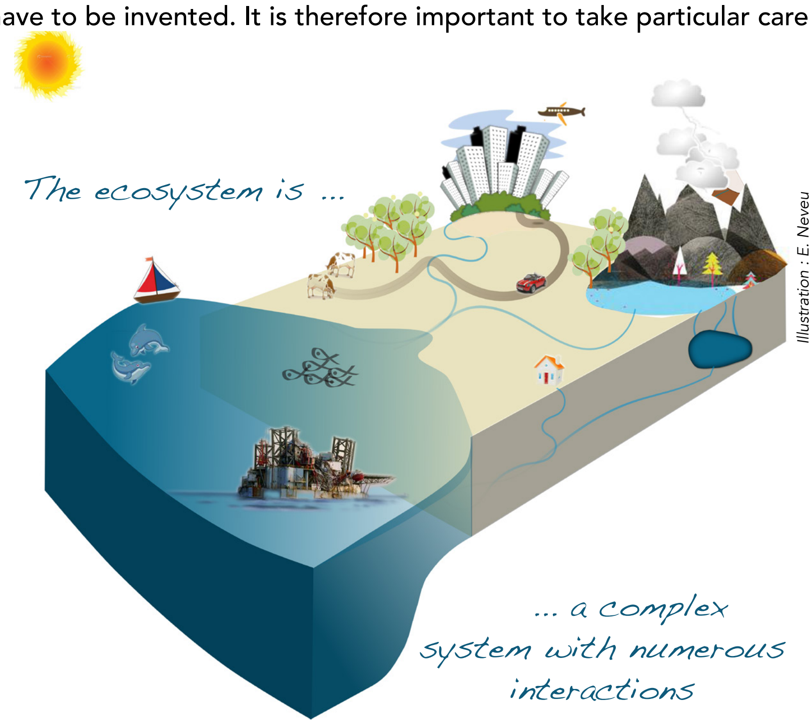
The French mathematics community has an enviable reputation on the world scene in almost every branch of the discipline. The great quality of its contributions is no small measure due to the importance of mathematics in French culture and training programmes, as well as the possibilities offered by full-time research posts such as those of the EPSTs (CNRS, INRA, INRIA, IRSTEA, IFFSTAR, IRD or INED), the EPICs (CEA, IFREMER, ANDRA) and the EPAs (SHOM).

"This type de post is rare, if not nonexistent, in other countries."
John Ball (Prof. Univ. Oxford), in ***Mathématiques, l'explosion continue***.

Mathematics in France, however, is more cloistered than elsewhere. The system of training and research units (UFRs) and the National Council of Universities (CNU) often acts as a brake on pluri-disciplinarity, unlike in the US or England where the links are stronger between mathematicians and physicists or biologists. In those countries, there are many research posts split between two institutes or departments as well as a number of small mixed groups. More mixing, but also more curiosity and enthusiasm: a climate that encourages discussion and ideas! But recent French initiatives are changing the situation. We cite several such initiatives in the text that follows, as well as in the report.

MATHEMATICS IN THE REAL WORLD

In this section we focus on mathematics as a tool for solving an environmental problem. Acting as an aid to understanding, forecasting or decision-making, mathematics is like a very practical toolbox. However, it does have some shortcomings: a few mis- or poorly-understood tools lie at the bottom of the box, some are still new because they are unusable, while others still have to be invented. It is therefore important to take particular care in maintaining the links between the real world, theory and computational science.



The point of view of theoreticians of other disciplines

Whether it is to understand a phenomenon or to answer questions asked by society or politics, the important thing is the question. For us, being creative is asking the right question and attempting to answer it. It's not important if the answer doesn't need the help of mathematicians (at first sight). But when we create and perfect our models, or when we adapt theories, we are in fact doing mathematics.

We spend a large part of our time in conceptualising a problem, in choosing or building appropriate models, in determining parameters, in understanding interactions between processes, in using data and experiments to compare results with reality, and in testing the limits of the model in order to better understand the mechanisms at work. You get the picture? Although we're not mathematicians, we are much more than simple "users"; we are "model makers". It is vital for us to have real interaction with mathematicians (See ***Emerging Mathematics*** and ***Numerical Mathematics***).

We are living in a world of increasingly rapid societal and environmental change. To meet new challenges resulting from this change, we need to take into account very complex systems, where the impact of human activities cannot be neglected and where the issues at stake are extremely heterogeneous, in that they are multi-scaled and involve multiple processes. But the environmental sciences[1] are like mathematics in that they still require, and will continue to require, fundamental research in order to better understand the processes at work, to "simplify" and formalise how we perceive the outside world. This is the work of certain theoreticians in physics, chemistry, biology, economics, and mechanics, theoreticians who build models. Although their approach is different (see adjacent insert *Point of view*), these model builders carry out work that is very close to mathematics. Links must be forged with mathematics so that problems can be taken into account more effectively. Indeed, progress in mathematics very often leads to progress in the applications. In the following paragraphs, you will find an overview of the different needs arising from theoretical research to the most applied.

Understanding thanks to theory

The need for mathematics becomes apparent especially when interactions have to be managed. For example, different time scales need to be taken into account in THEORETICAL ECOLOGY[2] in the case of ecological and evolutionary dynamics or for the representation of different scales of biodiversity (individuals, populations, species, communities...). The separation of scales, which is the method used up until now, is no longer possible with more refined processes. But at what quantitative scale can a macroscopic model be used to model a phenomenon? Or, conversely, from what scale must random fluctuations be taken into account? This same concern is also apparent in fluid dynamics, where modelling TURBULENCE IN GEOPHYSICS is still a difficult subject, particularly in cases that are strongly non-linear where the separation of scales is less obvious. As shown recently, randomness can play a very important role in describing bi-stable states over the long term.

Complex fluid flows, whether they concern gels or granular media, also exhibit behaviour that is difficult to model. Although such flows are particulate or granular, they cannot be treated by classical statistical physics, since they do not obey the second principle of entropy. Mathematics, which has made important contributions to statistical physics, could provide new theorems and new models. These studies are particularly important in that they could contribute to advances in all the fields mixing MICRO-MACRO scales, such as studies on collective movements.

Understanding phenomena also involves analysing data: knowing how TO ANALYSE MASSIVE QUANTITIES OF DATA or to infer models from limited data sets. For example, extracting the dynamics of coherent structures from satellite images requires that progress be made on dynamic optimum interpolation, image processing and data assimilation, as well as new geometrical analysis tools (see *Big Data p.7*). We feel a considerable need for interaction with

1 In this project, it was not possible to take into account astrophysics, internal geophysics and medicine. However, we would like to stress their importance.

2 Throughout the synthesis, this character style denotes the titles of sections in the report.

geometry or the calculus of variations, a need also present in themes from *GEOMORPHOLOGY* and the *MORPHOGENESIS AND GROWTH OF PLANTS*.

Observation and simulation of non-reproducible phenomena in the laboratory

Many studies require modelling and observing non-reproducible phenomena in the laboratory: *EVOLUTION OF ECOSYSTEMS*, of the *WATER CYCLE*, *DYNAMICS AND RESILIENCE OF NATURAL AND SOCIAL TERRITORIES*, *CLIMATE*, *OCEAN AND ATMOSPHERE*, *SOCIOLOGY OF NETWORKS*, *ECONOMY*. Two general themes run through these studies: *GLOBAL ENVIRONMENTAL CHANGE* and *COMPLEX SYSTEMS*. But mathematics is also frequently in evidence:

- **Coupling different processes:** should models be coupled, given that each model is specifically adapted to a process, or should the problem be considered as a whole, even if it means settling for a less "refined" model?

- **Taking heterogeneity into account,** whether this be the heterogeneity of agents in economic models, the different temporal and spatial scales of social dynamics, or the chemistry and physics in climate models. Spatial heterogeneity also raises problems of scale coordination in relation to the needs for mesh refinement and the coupling of local and global models.

- **Validating and mastering the models** with the ultimate aim of developing tools for conducting diagnostic analyses, indicating errors a posteriori, calibrating models, estimating uncertainty, analysing sensitivity, and analysing the viability and resilience of models.

Helping the governance of territories and the sustainable management of ecosystems

Increasingly, the modelling tool is used for forecasting and helping decision-making, whether this is in the *MANAGEMENT OF RADIOACTIVE WASTE*, the *MODELLING OF MARINE SOCIO-ECO-SYSTEMS*, for the *SUSTAINABLE MANAGEMENT OF RESOURCES*, or for understanding the mechanisms of *COOPERATION*.

With an urgent need for the development of numerical codes and their adaptation to operational algorithms, there is an insidious appearance of needs relating to numerical analysis (see ***Numerical mathematics***). But this is also leading to a real need for the formalisation of processes that are difficult to interpret intuitively. Thus, these issues relating to decision-making must take into the account the human element and involve both individual and collective decisions. The objectives are subject to change, sometimes poorly defined, and often multiple. It should also be noted that there is a need to simulate the dynamic aspect of ecosystems, societies and institutions. All these needs underline, above all, the necessity of creating links between the human and social sciences and mathematics.

Other examples such as hurricane Katrina, the earthquake in Aquila or the work of the IPCC also show that good communication is crucial between those who decide and those who have the expertise to carry out the decisions. In addition, to be able to better help decision-making, scientists need to have a better understanding of the uncertainties relating to simulations, and this will not happen without mathematics.

What is co-construction?

The day of conferences organised by IFREMER at IHP on 7 October 2013 focused attention on the need to build models by involving all the actors concerned.

Faced with complex systems that behave in a non-intuitive manner, decision-makers need to be able to estimate the impact of different management scenarios. However, it is not sufficient to provide them with estimates of uncertainties of each scenario to gain their confidence! From the outset, it is important to involve them in discussions and to work with them in constructing models. Co-construction contributes to a better understanding of the results of models by the decision-makers and also makes it possible to better integrate decision-makers' objectives and needs as well as the capacities of anticipation or feedback based on local knowledge.

Similarly, it is vital to model new processes, such as complex fluids, by associating mathematicians, physicists, and computational scientists in co-construction (see Proposal 4, p.10).

Working together: time and means

Through their needs, the applied sciences are instrumental in creating new mathematical theories. Yet many researchers regret there are not closer links with mathematicians, who are considered as "difficult to interest". The mathematical methods themselves are not widely disseminated towards applications, a situation that has led to frequent concern being expressed over the limited exploitation of research results. ANDRA, in particular, has stressed that "too little of its research conducted in collaboration with fundamental research becomes operational."

Several causes have been identified for this shortcoming. Communication is made difficult by differences in vocabulary and dialogue, but also in the centres of interest. In addition, three-year ANR projects are too short to establish a dialogue and identify a problem that interests the two (or even three) communities involved. Such collaboration requires time and effort and invariably a number of unsuccessful attempts. Added to this, there are also difficulties of evaluating projects and recruiting from several different disciplines. France is perhaps an exception, since in the US and England disciplines are not cloistered in Research and training units (UFR) and there is more enthusiasm and curiosity between the different experts. In France, the disconnection between disciplines is not conducive to mixing and interaction.

This being said, even though greater interaction encourages the creation of ideas, it should not be forgotten that it also takes effort and a certain non-conservatism to really work together. Each researcher must remain focused on his or her theme but a balance must be found between this need and the transversality of mathematics in applications. Bringing together individuals geographically and having a critical mass of diverse skills is a prerequisite to successful teamwork, but the first requirement is above all the willingness of people to work together and recognition of the community.

The transfer of methods to the operational sphere, even if that involves considerable developmental work, is vital for promoting the value of research. It is also important to make developments that are interoperable and reusable more available and to allow the scientific community access to data so as to improve the capitalisation of results. INRIA is a large structure that has done a lot in this regard, but there is a serious lack of smaller intermediate organisations, close to the university community. Here, it is worth mentioning the initiatives of the groups ALLenvi and ALListene, and the creation of simulation groups such as those in Paris and Grenoble (MaiMoSiNE). These are good ideas that should not be left on the shelf and must not be based solely on the dynamism of a few willing participants, likely to run out of energy in the long term. Decisions must be made on real directives and choices and the means to carry them out must be made available. The creation of permanent posts for administrative personnel and engineers will be an important step.

EMERGING MATHEMATICS

In this section we focus on emerging theories. New environmental and societal issues call for a better understanding of numerous complex and highly heterogeneous behaviour patterns. Inspired by these challenges, mathematicians are formalising new questions and attempting to provide answers (see Mathematics in the Real World) by digging deep into their mathematical culture. These answers sometimes require the development of new mathematics.

Many types of behaviour are still not adequately described by theory. For example, visco-elas-to-plastic fluids, the dynamics of populations, or cultural behaviour require new models. Other new challenges for mathematicians are the management of interactions, the analysis of massive data sets and the taking into account of randomness. Here we present the emerging theories, which address the problem of how to take into account heterogeneities, randomness and uncertainty.

Addressing heterogeneity

Two approaches may be adopted in dealing with complexity: one solution is to simplify by defining the invariants, thereby reducing the dimensions of the models or data; the second solution is to integrate more complexity so as to take into account all the interactions and all the scales. The two approaches are closely linked. Thus, one has to move toward complexity while at the same time keeping in mind the question being addressed, since, in taking a slightly different point of view, that which is complex may become more "simple".

In the first approach, based on simplification, the model reduction methods are focused on managing the large dimensions of models. Analysis of massive data sets requires knowing how to sort so as to construct relevant invariants with a view to analysing numerous and heterogeneous observations. For example, in *BIG DATA LOOKING FOR MATHEMATICS*[1], large dimension classification algorithms can, by aggregating a large number of very different, low-level indicators, answer complex questions.

The second approach aims to deal with problems in all their complexity, by focusing on integrating multi-scale or multi-process behaviour.

- **Multi-scale behaviour over time:** to understand the *EVOLUTION OF HUMAN BEHAVIOR*, a description is needed of interactions between cultural learning in the short term and genetic evolution in the longer term.

- **Multi-scale behaviour in space**: separating scales is sometimes difficult when there is strong interaction between the scales. This is the case in *MODELLING GRAVITY WAVES*, an exercise that suffers from a poor representation of energy cascades between scales, which are still poorly modelled. This is also the case in the *MODELLING OF ADAPTATION* where the ecology/evolution couplings must take into account not only local interactions but also impacts at the global scale.

- **Diffusion of energy** but also the spread of rumours or epidemics over a *COMPLEX NETWORKS*, whether it be social or biological. Introducing the effects of topology, taking into account non-homogenous diffusion, or assuming a multiple or even continuous state (as opposed to a binary healthy/sick condition, for example) leads to degrees of additional complexity and presents enormous mathematical challenges. Challenges are also found in the analysis of *COMPLEX FLUIDS*, such as avalanches or the flow of polar icecaps, with transitions of phase, free-surface instabilities or rupture fronts and dynamics.

- **Multi-processes:** a studying risk in river and coastal hydrology involves modelling the interaction between waves, sediments and the bottom. More generally, conditions at the edges and coupling conditions raise serious mathematical analysis problems. Finally, analysing multiplex networks capable of dealing with the combined spread of several epidemics, or multi-games taking into account several types of social interaction require using not just a single branch of mathematics but a mixture of different theories. We are thus dealing with a world in which determinism and stochastic processes cohabit.

Managing randomness

Dealing with randomness is indispensable in understanding behaviour at the scale of the individual or at the microscopic scale. However, being satisfied with discrete scales limits more in-depth studies of collective behaviour. In individual-based models, networks or games, the change to continuous models is taking place. In *GAME THEORY*, numerous developments, on idealised cases of repeated two-person zero-sum games, have made it possible move to continuous time and to make a link with an analysis of uniform behaviour. Progress is also being made on theories concerning the characterisation of *EMERGENCE AND SELF-ORGANISATION* phenomena. This requires renewing the tools of statistical physics, since here the second law of thermodynamics has not been verified. It is also difficult to express the laws of conservation, which could help describe behaviour at a macroscopic scale. However, there are some promising avenues to pursue, such as the combination of kinetic theory and game theory (theory of Mean

The point of view of mathematicians

Our work consists of formalising. It is sometimes difficult to get oneself accepted since, even if there are needs for formal mathematics (see ***Mathematics in the Real World***), these are not always priorities for the applied sciences. But our analyses are essential since they can be used to ensure that a model is exploited to its greatest potential or, on the contrary, to invalidate methods that intuitively appear correct. This does not always please everyone since it upsets customs and priorities. But basically all we want to do is to simply improve our understanding of phenomena.

Distrust and lack of confidence are also due to distance and the limited contact with other communities. It is the physicists, considered more likely to understand the vocabulary of other scientists, who are generally called on to create models. However, associating mathematicians from the outset in the construction of models would enable more innovative results to be obtained (see *Mathematics and the Earth: a long story* p.7).

1 Throughout the synthesis, this character style denotes the titles of sections in the report.

Illustration : Nasa

Field Games, see http://mpt2013.fr/les-jeux-a-champ-moyen/).

There is also the advent of DETERMINISTIC-STOCHASTIC MODELLING. In traditionally deterministic disciplines, stochastic processes are becoming more integrated with one another to better manage genetic randomness in population dynamics and to better QUANTIFY UNCERTAINTIES in numerous models such as those of climate systems. As for model reduction methods, these call for the use of determinism in stochastic methods. We are also seeing the development of stochastic partial differential equations in population dynamics. Could they be applied to other fields such as ... in subgrid scale parameterizations for fluid turbulence?

Managing an uncertain environment

An environment is uncertain because we have insufficient data available, as is the case for the topography of rivers or for the rheological laws. In these cases, we then use statistical approaches such as the STOCHASTIC METHODS FOR THE ANALYSIS OF RARE OR EXTREME VALUES. An environment may also be uncertain because it varies over time. Thus there are numerous problems where man intervenes with his changing objectives. We then seek to test the robustness or resilience of models in these environments, but also to predict the dynamics and the stationarities. In game theory, reduced games are used, in certain cases, to compensate for the multiplicity of balances. The structure of their balances then reveals the emergence of social norms. Research studies, taking into account temporal variability in the structure of the game, have also revealed changes within a sub-population as well as changes between sub-populations within the total population. Finally, the dynamic structure of networks is also envisaged. Thus, for example, percolation is studied in order to evaluate the robustness of a network when a certain number of nodes and links are removed. ■

Mathematics and the Earth: a long story

For several centuries, great mathematicians have been interested in our planet. P. Fermat studied the weight of the Earth, C.F. Gauss contributed to the development of geomagnetism and A. Tikhonov developed regularization techniques commonly used in geophysics. The work of H. Poincaré in celestial mechanics should also not be forgotten or that of L. Euler, J. R. D'Alembert, H. Navier and G. Coriolis which culminated in the Navier-Stokes equations with the Coriolis term, providing us with central elements of simulations in meteorology. The history of this equation is recounted with humour and a degree of irreverence by the comic strip, ***L'équation du millénaire***, which the FMSP distributes free on the internet (http://www.sciences-maths-paris.fr/fr/bd- 462.htm). In this section, we describe three other examples. It should be noted that the initiative ***"Un jour, une brève"*** is a much more exhaustive treatment (See *Further reading*, p 3).

- Living world: The stripes of the zebra or the polygons of the giraffe are among the most spectacular morphogenetic manifestations in the natural world. In 1952, the founder of information technology, A. Turing, proposed a system of two reaction-diffusion equations that mimed these structures. Then R. Thom was the first to mathematically define morphogenesis. More recently, problems of invasion in a heterogeneous milieu (periodic, random) and non-local interactions (competition between individuals of different genetic traits) have led to the study of integro-differential equations that sometimes exhibit very complex behaviour with multiple stationary states.

- Boundary layers: The boundary layer methods, initiated by L. Prandtl in 1904, divide the flow velocity field into an inner part and a part close to the edges. This approach was taken up by the oceanographers V.W. Ekman and W. Munk on very simplified geophysical models. Next came the development of the mathematical methods of singular perturbations after 1950, the date of the publication by K.O. Friedrichs. The literature in this field is already rich: P.A. Lagerstrom, J.D. Cole, M. Van Dyke, W. Eckhaus and J-L Lions. Important results on the degeneration of boundary layers were only published recently.

- Optimal transport: The theory of optimal transport was developed by G. Monge in the 18th century in the context of moving a pile of sand into a hole in the most economic way. This theory was then taken up again in 1940 by a mathematician, L. Kantorovich, for problems relating to optimal resource allocation. Other mathematicians finally succeeded in solving the problem, in the case of an infinite number of grains of sand, by formulating a continuous model. A little later, optimal transport was used as a powerful demonstration tool in EDPs, geometry and probability. The work of Y. Brenier, C. Villani et al. provides a perfect example.

must we follow the "big data" method?

Digitalisation of our world is resulting in ever-increasing amounts of data from social networks and citizen science (data collected by citizens). Substantial information technology needs are appearing with respect to data safety and storage, but also in mathematics! In the United States, laboratories such as MIT's Human Dynamics Lab bring together mathematicians, sociologists and IT specialists in a field referred to as "Computational Social Science", but in France there is no equivalent. Yet there is a real need to develop new theories not only in statistics and optimisation but also geometry (see the article by Stéphane Mallat in the Report). These theories could also be used in fields like geophysics where the amount of data to analyse (models and observations) continues to increase.

However, "big data", which is comparable to statistical approaches (individual-based, microscopic, multi-agent) counts for nothing without "small data". Small data, obtained from macroscopic and deterministic approaches, is used to understand phenomena and formalise the essential. This is the important role of fundamental research.

The mathematics to be developed is also to be found between the two, in the micro-macro and deterministic-stochastic mix. The approaches adopted by physicists working on these subjects (see ***Mathematics in the real world***) must mobilise mathematics.

NUMERICAL MATHEMATICS

Numerical mathematics maintains close links with theory. Going from one to the other is a daily sport for some mathematicians. Such mathematicians often focus on problems arising from applications: coupling of processes, model validation, parameterizations, high-performance computing (supercomputing) ... Forming a bridge between the abstract and the concrete, numerical mathematics has difficulties in finding its place in the world of research in France.

Numerical mathematics focuses on solving problems from other disciplines (see insert: Point of view of computational scientists). Thus problems often mentioned in the section ***Mathematics in the real world*** are also found in this section but addressed from a different viewpoint. The role of mathematics here is not only to make continuous theory more useful, but also to better define concrete problems by anchoring them in mathematical formalism. In this chapter we will see why this discipline "between disciplines" is a discipline in itself, by reviewing a number of different themes: numerical methods, data analysis, integrated systems approach.

Numerical methods

The characteristics of models depend a lot on applications. Thus, while *OCEAN-ATMOSPHERE MODELLING*[1] is characterised by constraints related to stratification, rotation or the turbulent character of flows (direct and inverse cascades of energy), the *MODELLING OF SHALLOW FLOWS* often tends to be constrained by physical effects still poorly described by models, such as erosion mechanisms, and numerous uncertain parameters, such as topography or the coefficients of the rheological laws. Models must evolve as knowledge about applications progresses. The advent of supercomputers, however, has naturally led to a race to improve resolution. The result is that numerous geophysical numerical models are now used outside their original field of validity or with inadequate forcings. Thus, at finer resolutions, certain effects such as non-hydrostatic effects can no longer be discounted, and the simplifying hypotheses of the models need to be re-examined. Modifying these hypotheses has a direct impact on the numerical methods to be used (different conservation properties and/or stability constraints). It should not be forgotten, however, that additional computing means could also be used to increase the complexity of numerical methods or to evolve towards statistical approaches!

The choice of discretization methods has an important impact on the physical solution of the models. For example, to manage the singularities at the Earth's poles in climate models, new approaches are considered, like the unstructured and icosahedral meshes or the systems of hybrid coordinates based on ALE (Arbitrary Lagrangian Eulerian) methods. To evaluate their potential, it is important to set up a systematic development of inter-comparison exercises in cases that range from the ideal to the operational so as to rationalise the different physical and numerical choices.

1 Throughout the synthesis, this character style denotes the titles of sections in the report.

The point of view of computational scientists

Numerical mathematics often brings together expertise in numerical analysis and information technology. As computational scientists, we are adept at computing, or at data processing and management. Because we constantly go back and forth between discrete and continuous theories, because we develop and analyse algorithms used by other disciplines, we are often at the interface between theory and applications. Our mathematics is carried out in co-construction.

Furthermore, we cannot simply define ourselves as computational scientists since we can also be analysts, information technologists, or even physicists, biologists or economists; we are often interdisciplinary. Indeed, we are like a bridge between the abstract and the concrete.

Data analysis

Statistics are used when questions are asked about data sets (observations and models), estimating sizes or parameters, comparisons, validation of scientific hypotheses, or comparing models[2] and measurements. Here are a few of the new *TRENDS AND NEW CHALLENGES IN STATISTICS FOR CLIMATE*, *THE ENVIRONMENT AND ECOLOGY*, or for the *EVOLUTION OF POPULATION GENOMICS*.

- **From data rarity to data abundance.** Current research is addressing, among other things, the selection of relevant covariables, the testing of scientific hypotheses, and multi-dependency due to heterogeneity. Thus, the abundance of data is leading to a need to refine the techniques used to extract available information, particularly by basing this in part on less abundant but more standardised data (see *Big data*, p.7). For this development, methodological reflection is needed on the optimality of dimension reduction techniques. In addition, there is a real need for more discussion and exchanges between computational scientists and statisticians.

- **From the local to the regional or global scale.** Multi-scale analysis is essential in climate and population studies and calls for research, for example, on the definition of relevant models of spatio-temporal and multivariate random fields. Another challenge involves getting the different scales and levels of organisation to communicate. The hierarchical modelling generally used is creating numerous needs with respect to inference algorithms and efficient and rapid simulations.

2 For a mathematician, the model designates the equations describing the system being studied; for a statistician, it designates all the choices leading to the probability distributions representing the variables under study; for a scientist, it designates the numerical model used to simulate the behaviour of the systems under study.

Integrated systems approach

- **Combining models and observations.** Recent developments in so-called *DATA AS-*

similation methods concern improving the way in which nonlinearities are taken into account and adapting methods to hierarchies, model coupling, and multi-scale observations (images, Langrangian data). We are aware of the need to enhance theoretical understanding and are encouraging closer collaboration with optimisation specialists. Finally, with a view to accelerating the implementation of territorial planning models and also to improve the models, data assimilation methods are being studied to estimate physical parameters and external forcings.

- **Validation and decision-making in the uncertain.** Two strategies may be envisaged in parallel. On the one hand, there is model simplification or reduction (simplified and interpretable models, sensitivity analysis, meta-models) that must provide the error between initial model and approximate model. On the other, a multi-model approach needs to be set up with a comparison and quantification of uncertainties of the different models, possibly by mixing determinism and stochastics (See ***Mathematics in the real world***). Thus, sensitivity analysis and quantification methods must be able to manage a large number of strongly correlated parameters, take into account any physical constraints, and manage interactions and feedback between models.

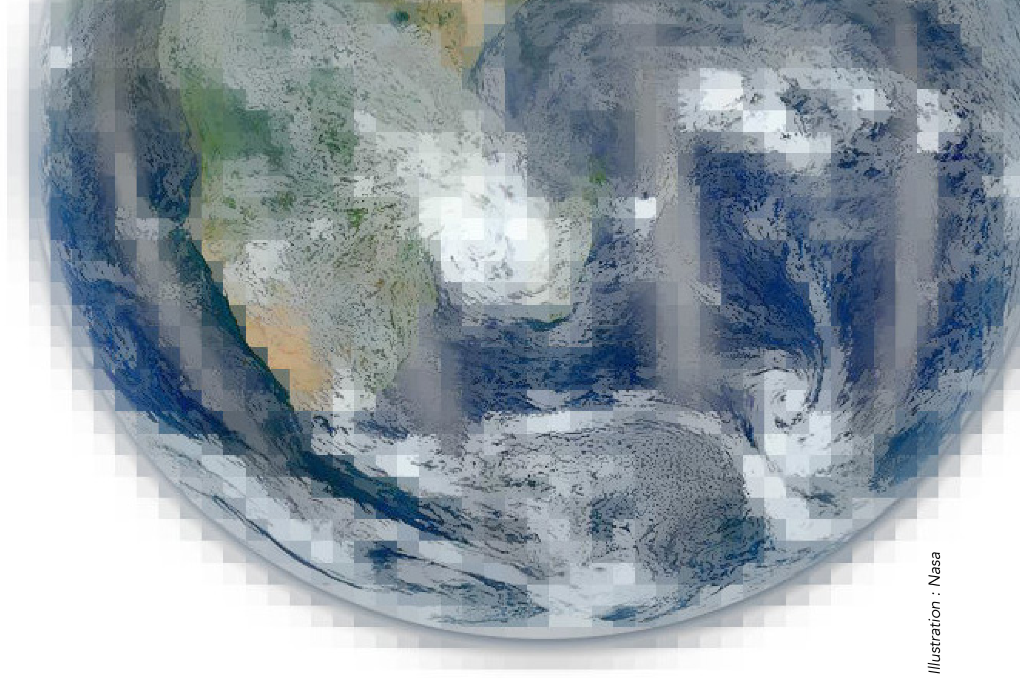


Illustration : Nasa

- **Coupling and interactions.** Boundary conditions or conditions at the interfaces have a strong impact on physical, numerical and mathematical quality as well as on the feasibility of the overall system. It is important to reflect on different strategies, take into account both intrusive and non-intrusive methods, and show interest in global-in-time methods limiting interactions between different processors and thus optimising the performance of computations on complex architectures. In geophysics, packages like AGRIF (Adaptive Grid Refinement In Fortran) or OASIS (Ocean, Atmosphere, Sea Ice, Snowpack coupler) are very popular. These approaches should be generalised! Finally, an integrated approach to system Earth cannot ignore man's impact and the way he adapts to change. These interactions, with complex feedback, call for better communication between applied mathematics, geosciences and human and social sciences in order to link decisional mathematics and sustainable development. An integrated vision of a system also requires setting up permanent posts (engineers, administrative personnel) to manage interaction between researchers and the promotion and exploitation of research findings (see *Working together*, p.5).

rethinking models for high-performance computing

Architectures evolve more rapidly than numerical methods and operational codes, which require several years of simulation and validation. However, we are seeing a real turning point with the increase in computing units and massively parallel processors. To be more efficient in terms of computing time, the codes can no longer be adapted simply.

Is substantial modification of the codes or methods inevitable? To answer this question, greater priority must be given to exploratory research: tests on simple methods and models can be used to check the feasibility and contribution of new techniques. From a mathematical point of view, there is a lot to be done on analysing coupled systems, multi-scale analysis, development of new methods, new grids and new algorithms. Numerous deterministic and stochastic models can only be redefined by treating a problem as a whole (maths/IT/application) and requires a communal effort from mathematicians, computational scientists, information technologists, and environmental scientists.

Furthermore, to validate exploratory research it is of course vital to support computing centres and help maintain operational codes.

Promoting work at the interface

To create real interaction, mathematicians must be placed in applied science laboratories since pure or applied mathematicians have preoccupations and objectives that are sometimes too far removed from applications. At the same time, it is important to maintain a high level in mathematics in order to be recognised by the community. This calls for a real effort from mathematicians.

These difficult careers are not given the value they deserve in France. In other countries, it is slightly different as there are small mixed groups and mathematics laboratories or departments, for example, in medical faculties. A brilliant student in mathematics will be encouraged to do a thesis at the interface between two disciplines, for example, in biology. In France, we are a long way behind and this is due in part to the very rigid system of the UFRs, the research and training units, and the problem of recruitment for and evaluation of interdisciplinary posts. However, there are facilities with the CNRS and the STII intersections, and with the new teams located at the Collège de France, particularly SMILE (Stochastic Models for the Inference of Life Evolution) led by Amaury Lambert, UPMC, at the CIRB*. We should also mention the LEFE/MANU programme (Fluid envelopes and the environment/ Mathematical and numerical methods) at INSU, the aim of which is to bring the geophysics and mathematics communities closer together.
These structures must be developed, maintained and extended to other "Maths and environment" issues.

** The CIRB (Center for Interdisciplinary Research in Biology) is a new Collège de France / CNRS / INSERM research structure located at the Collège de France, in the centre of Paris. Nine teams from different origins recently created this structure with the aim of promoting new collaborative agreements in biology and through its constituent disciplines.*

PROPOSALS FOR STRUCTURAL ACTION PLANS

The current state of mathematics research in France reveals a substantial community of researchers working on topics related to the Earth system. However, the links with other disciplines are still insufficient for a science related to the Earth that should be tending towards the development of a continuum, where there are no barriers between disciplines or within mathematics itself. We therefore propose some tools for financing research that are aimed at encouraging interaction and for which one of the essential selection criteria is the scientific quality of the mathematics questions addressed.

mathematics of planet earth axis

Given the action plan drawn up by the ANR, it is essential to have an important transverse axis, which we will call the "Mathematics of Planet Earth" (MPE), covering fundamental and applied research (see *Some challenges* p.12). This axis will have three objectives:

» *to make the link between the major societal challenges and the "all-knowledge challenge" on the MPE theme.* In the action plan, it is important to link fundamental research and applied research. This will encourage the development of new ideas and help new groups on topics that would not necessarily have been defined a few years beforehand.

» *to make the link between mathematics and the other disciplines.* It is important to explicitly mention mathematics (in the real world, emerging and numerical) in the action plan, and its importance in unlocking certain scientific doors. The essential criteria for selection must be the quality of the questions on the MPE theme as well as the quality of the multidisciplinary applicant group. Priority should be given to multilateral collaborative arrangements between a team of mathematicians and those from other disciplines, and to research focusing on a fundamental or applied MPE problem.

» *to make the link between French and international mathematics around the MPE theme.* Certain international actors (particularly UNESCO) have become aware that the Mathematics-Planet Earth link is very important for future research and therefore decided that MPE 2013 should be changed to "Mathematics of Planet Earth". The political actors such as Europe, the states and the regions have an important role to play in promoting the MPE theme.

Within this axis, we propose the following tools for financing, in addition to those already available. For all these tools, it is important to take special care in choosing the selection and evaluation juries to ensure that they are multidisciplinary.

DISSEMINATION OF INFORMATION

1 Summaries

Summaries are important if we wish to communicate with a wide audience about different issues, existing research methods and the models developed. The results of this work could be published officially by the CNRS or by a journal such as Maths In Action.

2 Intensive working group programme for young researchers

The aim of each working group, comprising mainly young researchers, will be to produce a deliverable such as summaries on a clearly defined multidisciplinary MPE subject. These intensive work groups could be officially located at the CIRM or the IHP. The CEMRACS (each year at CIRM) already organise sessions of this type but have difficulties in finding long-term financing.

3 Bilateral chairs

Propose a call for candidates for bilateral chairs for two teacher-researchers or researchers recognised in their respective disciplines who wish to make a long-term commitment to research in connection with MPE work, requiring expertise in two fields. In this context, we should not forget the availability of ERC synergy grants, the aim of which is to encourage the development of interdisciplinary groups. These calls for candidates should be intensified and continued at both the French and European level. In addition, the MPE theme and the involvement of several disciplines should be added as criteria for eligibility.

french initiatives

Multidisciplinary laboratories already exist in France and yet few researchers are aware of this! By way of example, we should mention the ENS in Lyon (Laboratoire Joliot Curie), the Mines de Paris (Chair in Prospective Modelling co-directed by Nadia Maïzi), the CNRS (Centre de Théorie et Modélisation de la Biodiversité in Moulis), or the Collège de France (CIRB, see p.9). These "project centres" welcome visiting research teams whose projects have been approved by an external scientific committee. The role of these centres is to develop, through close partnership with neighbouring laboratories, new experimental and conceptual approaches to address questions of multidisciplinary interest. They thus set up research and training programmes, organise working groups, and are able to welcome visiting researchers.

cooperative research

4 Fundamental and applied collaborative projects

Issue calls for candidates for MPE projects linking fundamental and applied research. These thinktanks must bring together researchers working on a precise subject covering the following aspects: modelling, mathematical analysis, numerical simulation. Teamwork will have to identify issues and bring together communities in the broad sense of the term. It will also have to reflect on the passage from algorithms based on simple codes to operational codes.

5 Project centres

Substantial financing structures must be put in place to officially create multidisciplinary teams and laboratories (with or without premises). These important projects require long-term financing, at least 2 x 5 years. It is worth pointing out that in other countries institutes of this kind already exist, such as the Santa Fe Institute. In France, however, the interesting initiatives that do exist (see *French initiatives* insert) are experiencing numerous difficulties in long-term financing and do not cover all the MPE topics.

nascent projects

6 MPE skills network

Propose a call for applications to promote skills networks related to the MPE theme, with the aim being to encourage contact between mathematicians and other disciplines. The creation of multidisciplinary Research and Training Groups (GdRs), an initiative to be encouraged, requires substantial financial means to organise research around a particular MPE theme. It should be noted that some multidisciplinary Research and Training Groups already exist (see the inside back cover) and that the ANR, the RNSC and the AMIES also offer similar possibilities, but without a clearly stated theme.

7 Nascent collaborative MPE projects

These calls for applications must enable multidisciplinary collaborative projects to be set up, focussing on MPE topics and supported by several tens of thousands of euros funding over a period of 4 or 5 years. These invitations for applications must be carefully defined and should give priority to new collaborative projects, such as the interaction of mathematicians and theoretical physicists in oceanography-meteorology. The PEPS at the CNRS level provide one possibility, but their duration is too short.

computational science axis

The sub-axis Foundations of computational science of the Society of Information and Communication challenge remains too focused on computational science, tending to overlook the importance of theory and making no reference to the societal issues of Planet Earth. It is important to show a clear preference for projects connected with fundamental mathematics and/or environmental and societal applications. We propose the following funding measures to enable the integration of computational science into MPE projects.

1 Ensure HPC resources and services

Intensive computations, whether for numerical simulation or the management of large data sets, are indispensable for the research described in this document. It is therefore of the utmost importance to continue financing the high-performance computing resources (including computation/data/network/expertise/support) of the French HPC ecosystem.

2 Provide long-term support for MPE Platforms

These platforms bring together mathematicians, computational scientists and researchers who address the major MPE challenges and adopt a holistic approach to issues: models-data-experimentation-methods-information technology. They may be in contact with environmental science observatories and located in modelling laboratories (for example, the Maison de la Simulation-Paris, MaiMoSiNE-Grenoble, CeMoSiS-Strasbourg or CaSciModOT-Orléans/Tours). A few platforms already exist but not many mathematicians are involved with them. The INRA, for example, has a platform on the Modelling of agricultural or forest ecosystems, while other modelling platforms are located at the Centre de Théorie et Modélisation de la Biodiversité (financed by the ANR) and the Institut Pierre Simon Laplace (Earth system modelling platform).

in the direction of other organisations

Alliances, institutes and universities are often divided internally on the basis of the topics studied, and fundamental research is at pains to find its place. Such a place, however, is essential. Fundamental and applied research should provide a transverse axis, cutting across applications. The representatives of this axis would cover randomness and determinism, theoretical and numerical aspects, and could be chosen through the learned societies. Thus, programmes linking institutes on MPE topics must be generalised. For this, inspiration must be sought from the LEFE/MANU at the INSU while at the same time associating it with the INSMI. Here, we put forward a number of proposals for institutes such as the CNRS, the universities and the Labex (Laboratory of excellence):

recognition of multidisciplinarity

1 Training in numerical analysis from the Bachelor's Degree

Since computing engineers are a rare commodity, it is important that students have at least the bases in numerical computation and information technology. For more specific needs in computational science, they will later have access to training in the doctoral schools and training centres.

2 Establish a quota of MPE bi-disciplinary thesis grants

These grants must be able to finance a thesis in its entirety. The doctoral student will be integrated in a collaborative MPE project and will be supervised by a teacher-researcher or researcher in mathematics and a teacher-researcher or researcher from another discipline.

3 Create engineers' posts shared between two teams

The engineer will make the link between a mathematics team and a team from another discipline working on MPE topics. The engineer will help promote and exploit research results, and will work to improve the distribution of information, methods and their use. He or she will also work on the development of generic tools and data accessibility.

4 Promote multidisciplinarity throughout the research career

Multidisciplinary activities are not appreciated for their just value either at the time of recruitment or during evaluations throughout a researcher's career. It is most important to ask questions in this respect, for example, when defining posts requiring interactions (see ***Mathematics in the Real World***, ***Emerging Mathematics, Numerical Mathematics)*** and when choosing members for selection or expert committees so that multidisciplinary activities can be taken into account throughout a career and during AERES assessments. It is therefore important to give priority to the creation of posts financed by Institutes where candidatures are eligible if, and only if, the work entails real interaction with other disciplines, and to increase the number of really multidisciplinary laboratories classified in more than one CNRS section, such as the GREMAQ in Toulouse where mathematicians mix with economists.

SOME CHALLENGES TO BE ADDRESSED

To conclude, we provide some examples of challenges relating to Planet Earth where mathematics has an important role to play. It goes without saying that this list not exhaustive. For a more comprehensive view, it is important to consult the Report as this provides a more complete list of the challenges, namely the sections on **Mathematics in the Real World, Emerging Mathematics** and **Numerical Mathematics**.

MPT2013 becomes Mathematics of the Planet Earth

The successful international year has now been extended to beyond 2013, a sign that the challenges linking mathematics and Planet Earth are important!

ocean-atmosphere

Analysis of major sets of satellite data
Ocean-atmosphere coupling, wind forcing
Internal waves: management of multiple scales, better parameterisation of sub-grids
Representation of sub-mesoscale processes for tracers and vertical transport in the ocean
Marine biogeochemistry: new coupling with population dynamics and large number of individuals

morphogenesis

Going from the cellular scale to a continuous model
Establishing the laws of scale that characterise the morphology of relief
Linking cellular and granular behaviour to a change in macroscopic form

ecosystems and biodiversity

Analysis of multiple interaction networks in the ecosystems
Taking localised interactions into account at the regional to global scale
Population and ecosystem dynamics in a changing and uncertain environment
Representations of different time scales in evolving eco-dynamics and biodiversity

climate

Land/atmosphere biogeochemical exchanges
Parameterisation of turbulence and the effect of clouds
Studies of sea ice and sudden changes of phase (solid, liquid, etc.) and dynamics
Rupture dynamics phenomena (polar icecaps, landslides, avalanches)

self-organisation

Understanding the disruptive phenomena of embryogenesis (fish, pollutants, etc.)
Importance of social mechanisms in the reproductive performances of animal populations
Characterising the phenomena of self-organisation (granular media, gels, social movements)

coastal flows

Marine energy forms
Resilience of structures to natural hazards
Better understanding of shoreline dynamics
Better understanding of wave breaking, erosion phenomena and sedimentation

river hydrodynamics and agro-ecosystems

Complex rheology flows
Better anticipation of natural risks
Water quality, infiltration, pollution
Coupling of river-stream-ocean models
Importance of bottom irregularities and problem of lateral confinement

economics and sustainable management of soils and resources

Linking cognitive mechanisms and evolution
Reducing the congestion of social, IT and transport networks
Management of spatial and temporal variability of energy forms
Towards a multi-model approach in the conception of strategies
Taking into account decision and anticipation mechanisms in helping decision-making
Modelling decision-making cognitive mechanisms and the influence of the environment on behaviour

MEETINGS AND CONFERENCES
APPROVED BY MATHSINTERRE RESEARCH THINKTANK (NON-EXHAUSTIVE LIST)
MANY INCLUDED ROUND TABLE DISCUSSIONS

FW: Fluid World
LW: Living World
HW: Human World

Journée de lancement MPT2013, Unesco, 5.3.2013, **Public lectures**

« Mathématiques pour la Gestion en Ecologie », 11.3.2013, Toulouse, **LW**

MATHBIO2013, 11-15.3.2013, Lyon, **LW**

Semaine des Mathématiques, Exposés pour lycéen(ne)s, 18-22.3.2013, **Public lectures**

Ecole du GdR EGRIN, 2-4.4.2013, Chalès, **FW HW**

ARP : Interfaces-Fluide, 17.4.2013, FW

« Mathematical and Computational Evolutionary Biology », **LW** 27-31.5.2013, Hameau de l'Etoile

SMAI2013, 27-31.5.2013, Seignosse Le Penon - Landes, **LW FW HW**

ARP : Théorie des jeux, 31.5.2013, HW

Journée Réseau National des Systèmes Complexes, 7.6.2013, Paris, **HW**

ARP : Développement durable, 4.7.2013, LW

CEMRACS 2013, 22.7-30.8.2103, CIRM Marseille, **Public lectures**

« Fluides complexes en glaciologie et glissements de terrain », 9-13.9.2013, Toulouse, **FW**

6ème Journée des Méso-centres, 19.9.2013, Paris

Journée Maths pour la Terre, 19.9.2013, Toulon-Var.

ARP : Turbulence et paramétrisation, 24.9.2013, FW

« Ondes de gravité en domaine côtier et fluvial », 30.9.2013, Bordeaux, **FW**

METEO, 30.9-1.10.2013, Orléans, **HW**

ARP : Journée IFREMER, 7.10.2013, FW LW HW

« Transports en sous-sol », 17.10.2013, La Rochelle, **FW LW**

MCPIT 2013, GdRE ConEDP, 18-22.11.2013, Paris, **FW**

Entretiens Jacques Cartier, 25-26.11.2013, Lyon, **FW LW**

Non-exhaustive list: consult the Report for a complete list.

INVITATIONS
FOREIGN RESEARCHERS
OUTSIDE CONFERENCES

Hugh Possingham
(Univ. of Queesnland)
Mathematics and Ecology management
11 March 2013, Toulouse

Karl Sigmund
(Faculty of Mathematics, Vienna)
Manfred Milinski
(Max-Planck Institute, Plön)
Théorie des jeux et contrats sociaux,
31 May 2013, IHP

Simon Levin
(Princeton Univ.)
Développement durable et mathématiques,
4 July 2013, IHP

Rupert Klein
(FU Berlin)
Maths and géophysics
5 April 2014, Chambéry

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NETWORKS

Research Groups, GDR-INSMI
calcul scientifique Calcul, Loïc Gouarin
contrôle des edps ConEDP, Fatiha Alabau
écoulements gravitaires Egrin, Stéphane Cordier
théorie des jeux Jeux, Jérôme Renault
analyse stochastique Mascot-Num, Clémentine Prieur
gestion des déchets nucléaires Momas, Tony Lelièvre

Networks
maths-entreprises AMIES, George-Henri Cottet
systèmes complexes RNSC, Guillaume Déffuant

SUPPORTS
FROM INSTITUTIONS

Mathematics of Planet Earth *2013*
Christiane Rousseau, Mireille Chaleyat-Maurel
Mathematics of Planet Earth : mpe2013.org
Un jour, une brève : mpt2013.fr

Academic Associations
SMAI, Thierry Goudon
SMF, Cyril Imbert
SFdS, Liliane Bel

Popular Science
Interstices, Jocelyne Erhel

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Mathématiques en Interactions pour la Terre

The MathsInTerre Research Thinktank of 2013 is a response to the call for proposals for discussion and reflection on the topic "Mathematics and the Complexity of the Earth System" issued by the ANR (French National Research Agency) in 2012. This summary, the result of multidisciplinary interaction throughout 2013, is based on the more complete Report organised in the form of information sheets. It appeared to us necessary to first remind readers that since mathematics is a varied and transverse discipline, mathematicians must be flexible and open to the ideas of others. In particular, when we expose new needs and present new mathematical theories, we stress the importance of focusing on the Science-Earth link as a continuum, without any barriers between disciplines. It is the questions arising from concrete issues that must govern the type of mathematics conducted for studies of the Earth system, whether it is mathematics in the real world, emerging mathematics, or numerical mathematics. Thus one of the essential criteria in determining the quality of applied mathematics must be the scientific quality of the questions asked. The document concludes with proposals for structural actions that must encourage interaction with other disciplines, such as human and social sciences, life sciences and earth sciences.

This brochure was prepared as part of the MathsInTerre Research Thinktank 2013

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With the support of
Agence Nationale de la Recherche - French National Research Agency
CNRS - French National Center for Scientific Research
Henri Poincaré Institute
Université de Savoie

and the support of the Fédération de Recherche en Mathématiques Rhône-Alpes-Auvergne